

University of Illinois at Urbana-Champaign Department of Physics

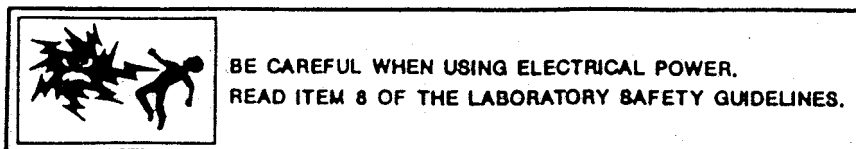
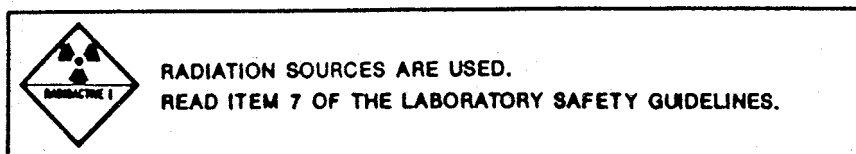
Physics 401 Classical Physics Laboratory

### Experiment 54

## Measurement of the Electronic Charge by the Oil Drop Method

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## I. References

1. T. B. Brown, editor, *The Taylor Manual of Advanced Undergraduate Experiments in Physics*, p. 392, Addison-Wesley, 1959.
2. R. A. Millikan, *The isolation of an ion, a precision measurement of its charge, and the correction of Stokes's Law*, Physical Review, 32, p. 349, 1911.
3. Harvey Fletcher, *My work with Millikan on the oil drop experiment*, Physics Today, p. 43, June, 1982.
4. I. T. Lee *et al.*, Large bulk matter search for fractional charge particles, Phys. Rev. D 66, D012002, 2002, and M. L. Perl *et al.*, *Search for isolated fractionally charged particles*, <http://www.slac.stanford.edu/exp/mps/FCS/FCS.htm>.
5. Portions of this lab writeup come from PASCO Manual 012-06123D, available at <http://www.pasco.com>, and are copyrighted by PASCO, reproduced here with permission.

**Note:** The radioactive source used in this experiment is Thorium-232, a naturally occurring, low-level alpha particle emitter with a half-life of  $1.41 \times 10^{10}$  years. The strength of the source is 0.008 mcurie. It is not regulated and poses no hazard to the user of the PASCO Millikan Oil Drop Apparatus.

## II. Introduction

One of the most important physical quantities is the magnitude of the electronic charge,  $e$ . The first precision measurement of the value of  $e$  was accomplished by the American physicist, Robert A. Millikan (1868-1953), who in 1911 reported the results of his oil drop experiment, done at the University of Chicago.

In this experiment a charged oil drop is introduced between two oppositely charged horizontal plates where its velocity of fall under gravity and its velocity of rise in response to a suitable electric field are measured. From these data, the charge on the droplet may be calculated. It is found that all such charges are integral multiples of a smallest charge. A modern version of the oil drop experiment ran recently at SLAC in a search for fractional charges for. See reference 4.

## III. Theory

An oil drop with charge  $Q$  and mass  $m$  moves vertically up or down through air, which acts as a viscous medium. The four forces on the oil drop are (1) the force of gravity, (2) the drag force

through the air, (3) the electric force from the electric field (when the electric field is on), and (4) the buoyant force from the air displaced by the drop. The force of gravity is always down. The drag force opposes the motion of the drop and may be either up or down, depending on the direction of the drop. The electric force depends on the sign of the charge,  $Q$ , and the direction of the electric field,  $\vec{E}$ , and may be up or down. (In our experiment the electric field is used to pull the drop up so the electric field direction is chosen to make the direction of the electric force up.) The buoyant force is always up and is simply taken into account by using the difference in the densities of the drop material, oil, and the fluid which it displaces, air. We use below  $\rho = \rho_{oil} - \rho_{air}$ . (This effect is small and is included for completeness and historical authenticity.) We put the  $z$  axis of our coordinate parallel to the up and down directions, and we choose up to be in the positive  $z$  direction. The force of gravity on a drop of mass  $m$  is then  $\vec{F}_g = -mg\hat{z}$ , where  $g$  is the gravitational constant,  $9.801 \text{ m/s}^2$ , and  $\hat{z}$  is a unit vector in the  $+z$  direction.<sup>1</sup> The drag force on a drop of radius  $a$  from Stokes' law is  $\vec{F}_{drag} = -6\pi\eta a \vec{v} = -K \vec{v}$ . The constant  $\eta$  is the viscosity of air, and a (positive) constant  $K$  is introduced for convenience.<sup>2</sup> The minus sign shows that the drag force is in the opposite direction from the velocity of the particle. If the particle is falling and  $\vec{v}$  is in the  $-z$  direction, the drag force is in the  $+z$  direction.<sup>3</sup> The electric force is  $\vec{F}_E = Q\vec{E}$ , where  $Q$  is the charge on the particle<sup>4</sup> and  $\vec{E}$  is the electric field. The electric field can either be in the  $+z$  direction or  $-z$  direction, depending on whether the potential difference between the lower plate and the upper plate,  $V_{lower} - V_{upper}$ , is positive or negative, respectively.

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<sup>1</sup> If we were not using vectors, we would simply write  $F = -mg$ , and the minus sign denotes that the force is down.

<sup>2</sup> We must make a small correction to Stokes' Law, the Cunningham correction, because of the size of our drops. Millikan also had to make this correction. With this correction the equation reads  $\vec{F}_{drag} = -(6\pi\eta a/f_c)\vec{v}$ . We simply need to redefine the constant,  $K$ , in Eq. 3 below.

<sup>3</sup> If we were not using vectors, we would say "if the particle is falling (rising), the velocity is negative (positive) and the drag force is positive (negative)," i.e.  $F_{drag} = \mp (6\pi\eta a/f_c)v$ .

<sup>4</sup> The charge  $Q$  could be either positive or negative, depending on whether the electrons have been removed from or added to the drop.

Having identified the forces on the particle, Newton's law is then

$$\vec{F} = m \frac{d\vec{v}}{dt} = \vec{F}_g + \vec{F}_{drag} + \vec{F}_E. \quad (1)$$

When the particle reaches terminal velocity,  $\frac{d\vec{v}}{dt} = 0$ , and Newton's law becomes

$$\vec{F}_g + \vec{F}_{drag} + \vec{F}_E = 0. \quad (2)$$

(See the appendix for a calculation of the time for the particle to reach terminal velocity.) With no electric field and using the expressions for the gravity and drag forces, Eq. 2 gives

$$K \vec{v}_g = -mg\hat{z}, \quad (3)$$

the particle is falling and the velocity is in the  $-z$  direction.  $\vec{v}_g$  is the velocity of the (falling) drop on which the forces are gravity and drag. The drop is spherical due to the surface tension of the oil, and we can relate the mass to the radius of the drop:  $m = 4\pi \rho a^3/3$ . In effect, we will measure the velocity of the falling drop to determine its mass and its radius. With an electric field and using expressions for all three forces, Eq. 2 becomes

$$Q\vec{E} = K(\vec{v}_E - \vec{v}_g). \quad (4)$$

$\vec{v}_E$  is the velocity of the drop on which the forces are gravity, drag, and electric. We now can write an equation for the charge  $Q$ . In principle,  $\vec{E}$  and  $\vec{v}_E$  could be in either direction. In practice, we only use data for which  $\vec{v}_E$  is in the  $+z$  direction. For these data we may write  $\vec{v}_E = v_{rise} \hat{z}$ , where  $v_{rise}$  is the speed of the rising drop.<sup>5</sup> We may also write  $\vec{v}_g = -v_g \hat{z}$  where  $v_g$  is the speed of the drop falling under the force of gravity. The equation for  $Q$  becomes  $Q\vec{E} = K(v_{rise} + v_g)\hat{z}$ . The numerical factor on the right hand side is positive so  $Q\vec{E}$  must be in the  $+z$  direction. Thus, when observing a rising drop, if the electric field is in the  $+z$  direction,

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<sup>5</sup> Speeds are positive.

$Q$  is positive, and if the electric field is in the  $-z$  direction,  $Q$  is negative. A complete, but, perhaps, not the simplest, equation for the charge is

$$Q = \pm \frac{K (v_{rise} + v_g)}{|\vec{E}|} \left\{ \begin{array}{l} + \text{for } E \text{ in } +z \\ - \text{for } E \text{ in } -z \end{array} \leftrightarrow \begin{array}{l} V_{lower \text{ plate}} > V_{upper \text{ plate}} \\ V_{upper \text{ plate}} > V_{lower \text{ plate}} \end{array} \right\}. \quad (5)$$

There should be no ambiguity in this equation.  $|\vec{E}|$  is the (positive) magnitude of the electric field. The sign of  $Q$  is given by the direction of the electric field as noted. Writing out explicitly the factors in  $K$ , Eq. 5 becomes

$$Q = \pm \frac{6\pi\eta a (v_{rise} + v_g)}{f_c |\vec{E}|} \left\{ \begin{array}{l} + \text{for } E \text{ in } +z \\ - \text{for } E \text{ in } -z \end{array} \leftrightarrow \begin{array}{l} V_{lower \text{ plate}} > V_{upper \text{ plate}} \\ V_{upper \text{ plate}} > V_{lower \text{ plate}} \end{array} \right\}. \quad (6)$$

This equation is not yet useful for determining  $Q$  because it contains the drop radius and the Cunningham correction factor. We determine the drop radius,  $a$ , from Eq. 3. Using Stokes' Law and the equation for the mass of drop, we obtain from Eq. 3,

$$(6\pi\eta a / f_c) v_g = (4\pi \rho a^3 / 3) g. \quad (7)$$

The Cunningham correction factor,  $f_c$ , complicates the determination of the radius of the drop considerably, because the correction itself depends on the radius of the drop. We show in the appendix that to a good approximation the Cunningham correction is

$$f_c = 1 + r_c / a. \quad (8)$$

In this expression  $r_c$  depends on constants and the ambient pressure. This expression is discussed in the appendix where the numerical expression

$$r_c [\text{m}] = \frac{6.18 \times 10^{-5}}{p [\text{mm Hg}]} \quad (9)$$

is justified. (The ambient pressure is entered in mm Hg, and the radius correction constant is obtained in meters.)

Let us write a version of Eq. 7 which does not contain  $f_c$  and in which we introduce,  $a_o$ , the uncorrected drop radius. This version is useful, since, in our experiment, the correction is small.

$$6\pi\eta a_o v_g = (4\pi \rho a_o^3 / 3)g . \quad (10)$$

Solving Eq. 10 for the uncorrected drop radius,  $a_o$ , we obtain

$$a_o = \sqrt{\frac{9\eta v_g}{2g \rho}} . \quad (11)$$

The speed of the falling drop,  $v_g$ , gives the uncorrected drop radius,  $a_o$ , directly. Using  $a_o$  and  $r_c$ , Eq. 7 (the equation for the drop radius) can be written as

$$a = \sqrt{\frac{1}{f_c}} a_o = \sqrt{\frac{1}{1 + \frac{r_c}{a}}} a_o \rightarrow a^2 + r_c a - a_o^2 = 0 . \quad (12)$$

We choose the positive root of the quadratic equation, and we find for the drop radius

$$a = \sqrt{a_o^2 + (r_c/2)^2} - r_c/2 \approx a_o - r_c/2 = \sqrt{\frac{9\eta v_g}{2g \rho}} - \frac{r_c}{2} . \quad (13)$$

We note again that the speed of the falling drop,  $v_g$ , gives the (corrected) drop radius,  $a$ , directly. The approximation is justified in the appendix.

It is interesting to calculate the radius of the drop,  $a$ , but the expression for  $Q$  contains the ratio  $a/f_c$  and not  $a$ . Using  $a/f_c = a_o/f_c^{3/2}$  from Eq. 12 and Eq. 11, we obtain a more useful expression for  $Q$ ,

$$Q = \pm \frac{6\pi\eta}{f_c^{3/2}} \sqrt{\frac{9\eta}{2g\rho}} \sqrt{v_g} \frac{(v_{rise} + v_g)}{|\vec{E}|} \left\{ \begin{array}{l} + \text{for } E \text{ in } +z \\ - \text{for } E \text{ in } -z \end{array} \leftrightarrow \frac{V_{lower \text{ plate}} > V_{upper \text{ plate}}}{V_{upper \text{ plate}} > V_{lower \text{ plate}}} \right\} . \quad (14)$$

For the electric field we write  $|\vec{E}| = V/d$ , the voltage difference between the plates,  $V = V_{\text{lower}} - V_{\text{upper}}$ , divided by their separation,  $d$ . In our experiment the speeds are measured by timing the drop over a fixed distance  $x$  so for  $v_g$  and  $v_{\text{rise}}$  we use  $v_g = x/t_g$  and  $v_{\text{rise}} = x/t_{\text{rise}}$ , respectively. The final expression for the charge on the drop is then

$$Q = \left[ \frac{1}{f_c^{3/2}} \right] \left[ \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \right] \left[ \sqrt{\frac{1}{t_g} \left( \frac{1}{t_g} + \frac{1}{t_{\text{rise}}} \right)} \right]. \quad (15)$$

Note that the sign of  $Q$  is given by the sign of  $V$ . All other quantities in Eq. 15 are positive. The expression for  $Q$  is a product of three terms, denoted by the brackets. The first term depends on the Cunningham correction factor; the second term is common to all drops (since we can choose to use the same  $d$ ,  $V$  and  $x$  for every drop); and the third term is unique to the individual drop.

We will use the data to test the quantization of charge; if charge is quantized, we may write  $Q = ne$ , where  $n = \pm 1, \pm 2, \dots$ . The charge of the electron is with this definition  $-e$ , and  $e$  is a positive number. Thus we can make a determination of  $ne$  for every drop for which we make fall and rise time measurements. The physical constants in Eq. 15 are given in the table below.

quantity	Value
Viscosity of air	$\eta = 1.8478 \cdot 10^{-5} \text{ kg/m}\cdot\text{s} \text{ (25 }^\circ\text{C)}$ $\Delta\eta/\Delta T = 0.0048 \text{ kg/m}\cdot\text{s}/^\circ\text{C}$
Density of oil	$\rho_{\text{oil}} = 886 \text{ kg/m}^3$
Density of air	$\rho_{\text{air}} = 1.29 \text{ kg/m}^3$
Acceleration due to gravity	$g = 9.801 \text{ m/s}^2$

#### IV. Experimental apparatus

The experimental apparatus is shown schematically in Figure 1 and in the diagrams of Figures 2 through 4. P1 and P2, shown in Figure 1, are two carefully aligned, conducting, circular horizontal plates; the upper one has a small hole on the bottom to permit the oil droplets to enter the region between the plates. The region between the plates is viewed by a long-range microscope, whose eyepiece contains a graduated scale in two dimensions, called a reticle. The major tick marks in the graduated scale mark distances of 0.5 mm; the minor tick marks

correspond to distances of 0.1 mm. Light for viewing the drops is provided by a 5 Watt halogen bulb, with a dichroic, infrared heat-absorbing window. Horizontal and vertical adjustment knobs are provided to change the position of the filament in order to obtain the best illumination of the droplets in the viewing chamber. We will be using a small video camera to view the drops through the microscope. The image from the camera is sent to the computer at the lab station, and shown on the screen.

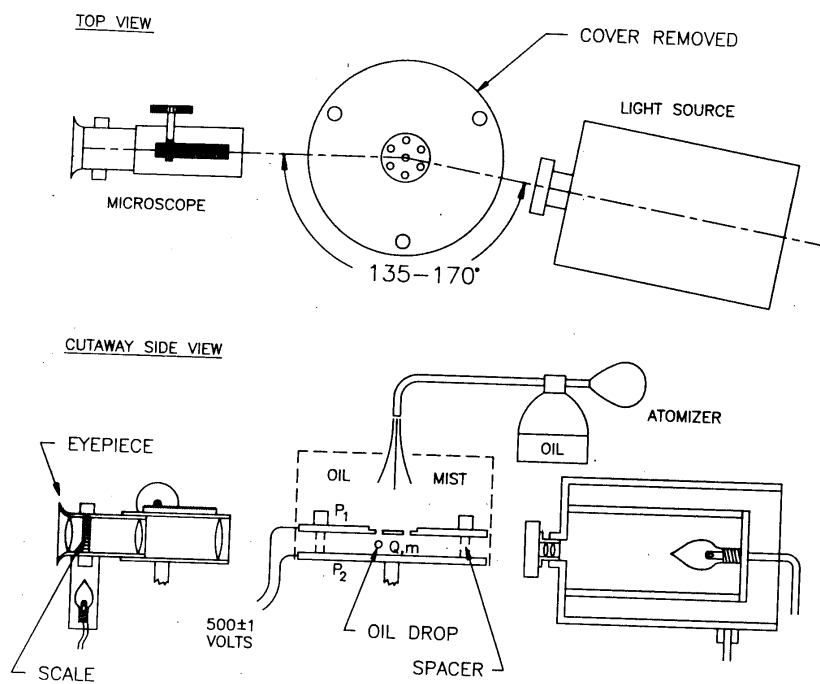


Figure 1. Millikan oil drop apparatus (schematic).



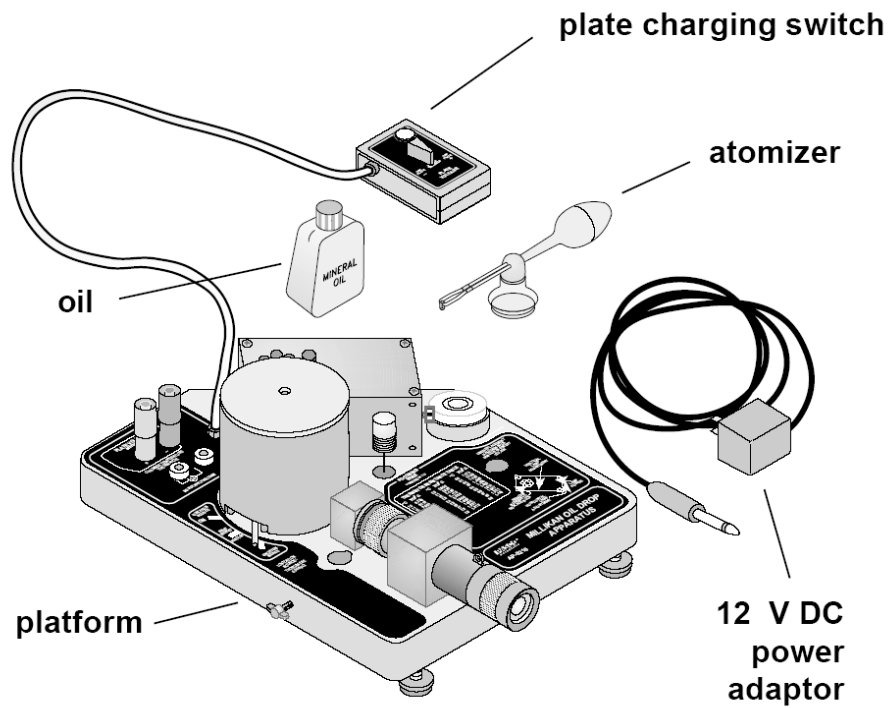


Figure 2. PASCO Millikan oil drop apparatus.

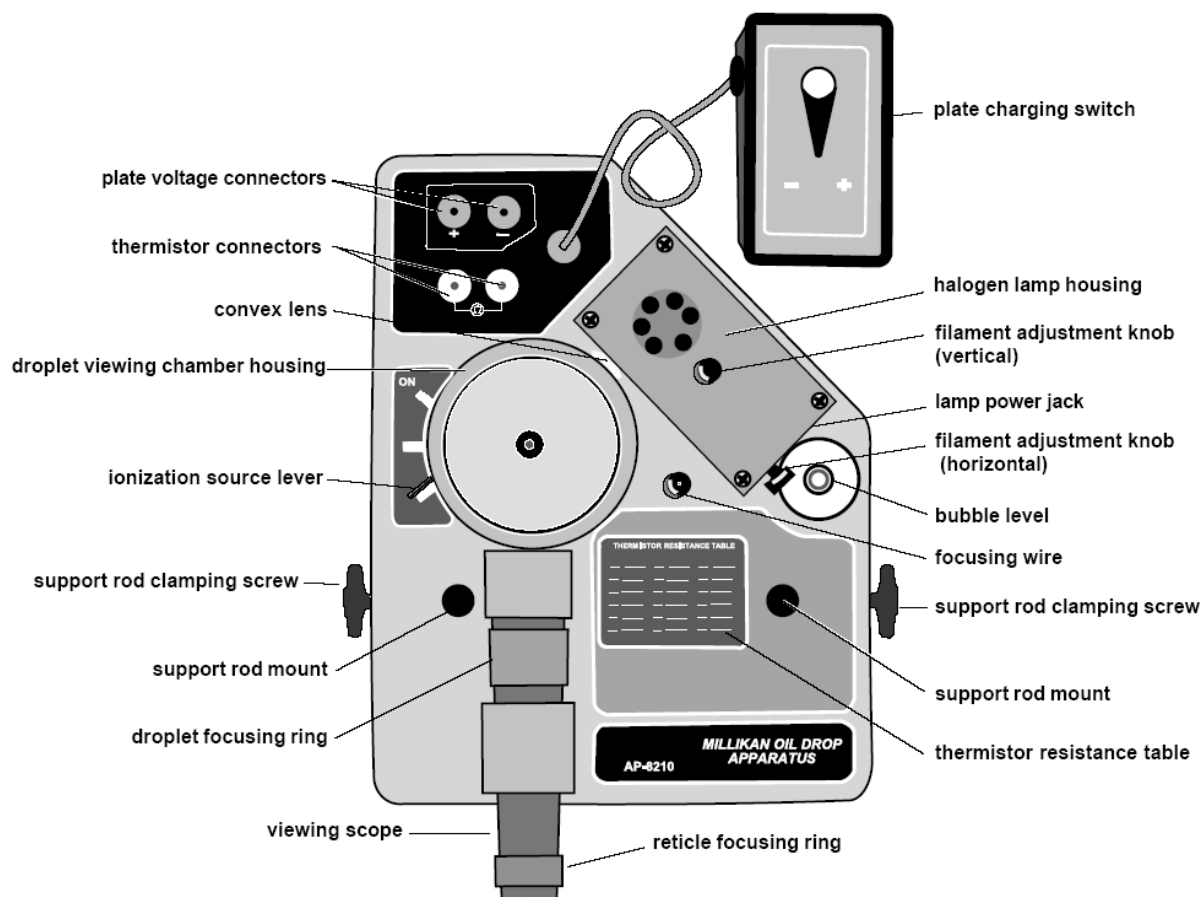


Figure 3. PASCO Millikan oil drop apparatus, top view.

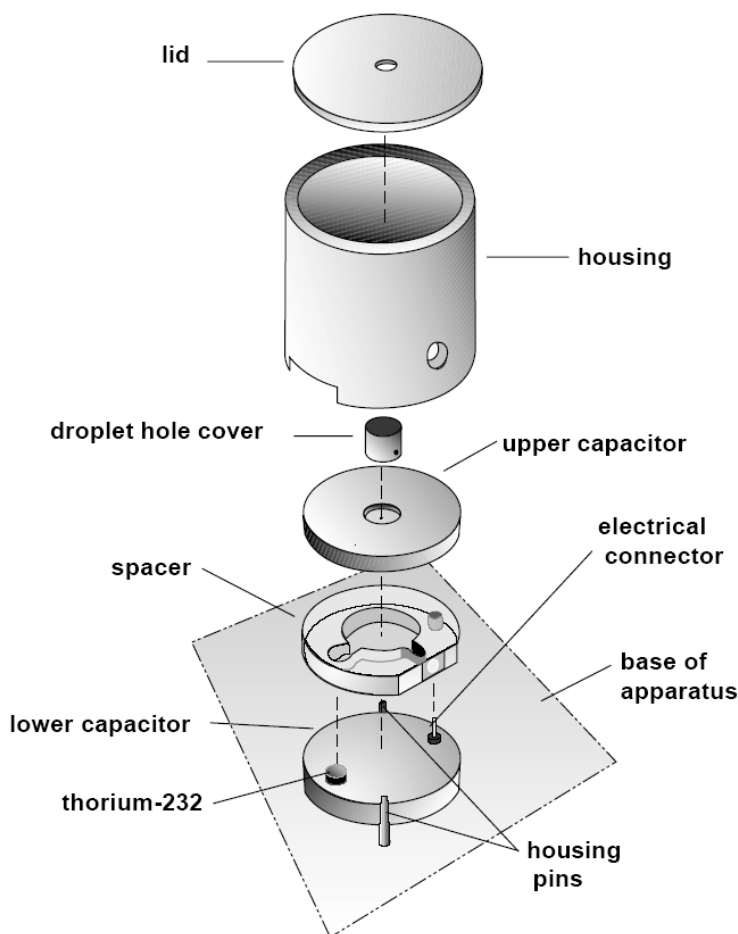


Figure 4. Droplet viewing Chamber

### V. Data acquisition program

Data acquisition is facilitated with the LabWindows/CVI program Timer1.prj. A shortcut to the program should be on the PC desktop. The program assists in the measurement of rise times and fall times and logs the measurements to a file. (You may record the data simultaneously in your notebook, if desired.) If there are difficulties with the program, a digital timer is available as a backup. To use the program, turn on the computer and log in. Double click on the desktop shortcut and then load the program 'Timer1.prj'. The program has 'Help' available at the pull-down menu. You should become familiar with each button in the graphical interface before proceeding with your data taking.

The timer program works both with the green board and the yellow board. The green board has an electronic timing circuitry and is connected with the Lab-PC+ data acquisition card inside the computer. The yellow board has 'start' and 'stop' buttons for you to time the oil drops. Make sure to move the flat ribbon cable at least one foot away from the black power cable. Crosstalk

between those two cables may initiate a counting sequence when 500 volt polarity switch is activated, if they are too close to each other.

Analysis of the data should be done with Origin. The output of the timer program is in Excel format which can be readily imported into Origin using the import Wizard. In addition to the timing data, you will need to enter other data such as pressure, plate spacing etc. Take some time to look at the analyzed data.

## VI. Procedures for measuring the charge on a drop

1. **Record the inventory number on the base of your apparatus.**
2. **Determine the barometric pressure in mm Hg** at the start and finish of each laboratory period. An aneroid barometer is located on the tool board on the west wall of the lab. After tapping the instrument gently, read the black needle. The red needle is just a pointer and should be ignored. You will need the temperature of the air in the apparatus, but this is measured by a thermistor in the droplet viewing chamber. The pressure and temperature are needed in the data analysis.
3. **Measure the plate separation:** Disassemble the droplet viewing chamber by lifting the housing straight up and then removing the upper capacitor plate and spacer plate. (See Figure 4). Measure the thickness of the plastic spacer (which is equal to the plate separation distance  $d$ ) with a micrometer. Be sure that you are not including the raised rim of the spacer in your measurement. This measurement is important to determine the electric field and appears in the expression for the measured charge. Record your measurement. *Use care when handling the brass plates and the plastic spacer in order to avoid scratching them. All surfaces involved in the measurement should be clean to prevent inaccurate readings.*
4. **Align the optical system and focus the viewing scope:** Reassemble the plastic spacer and the top capacitor plate onto the lower capacitor plate. Replace the housing, aligning the holes in its base with the housing pins (see Figure 4). The thorium source and the electrical connection on the lower capacitor plate fit into appropriately-sized holes on the plastic spacer. Unscrew the focusing wire from its storage place on the platform and carefully insert it into the hole in the center of the top capacitor plate (Figure 5). Be careful not to bend the focusing wire.

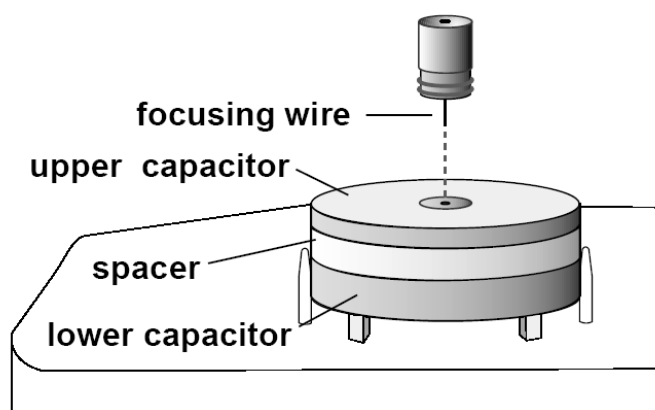


Figure 5. Insertion of the focusing wire into the top capacitor plate.

Connect the 12 V DC transformer to the lamp power jack in the halogen lamp housing and plug it into a 120 V AC socket. Bring the reticle into focus by turning the eyepiece-reticle focusing ring. View the focusing wire through the viewing scope, and bring the wire into sharp focus by turning the droplet focusing ring. Note: If you wear glasses, viewing might be easier if you do not use your glasses but instead adjust the reticle focusing ring. Adjust the halogen filament position with the horizontal adjusting knob on the halogen lamp housing. The light is best focused when the right edge of the wire is brightest (in highest contrast compared to the center of the wire). While viewing the focusing wire through the viewing scope, turn the vertical filament adjustment knob until the light is brightest on the wire in the area of the reticle. Return the focusing wire to its storage location on the platform.

5. **Use the bubble level attached to the experiment base to level the platform** by adjusting the screws on the rod stand base. These screws should be seated in small brass coasters. If during the experiment the drops seem to “walk” sideways, leveling should be redone.

6. **Attach high voltage cables and reassemble the droplet viewing chamber.** With the plate charging switch set to “PLATES GROUNDED”, connect the high-voltage cables from the power supply box to the plate voltage connectors on the Millikan apparatus platform. The power supply box is located over two transfer-circuit binding posts on the wall). The high-voltage supply is in series with an isolation resistor in order to protect you from a possible 500-volt shock. Do not tamper with the power supply box. The voltage on the transfer circuit comes from an accurate, stable, regulated power supply and is maintained at  $500 \pm 1$  volts. The isolation resistor in no way alters this voltage, since no current is drawn by the apparatus. Reassemble the droplet viewing chamber by placing the droplet hole cover on the top capacitor plate and then placing the lid on the housing (See Figure 4).

7. **Measure the thermistor resistance**, which provides a measurement of the temperature of the air inside the droplet viewing chamber. The relationship between the thermistor resistance

and temperature is written on the platform, and is reproduced in Appendix I. The thermistor is in the lower brass plate and should correspond to the temperature of the droplet viewing chamber.

8. **How the controls work:** There are two controls – the ionization source lever, and the plate charging switch.

When the ionization source lever is in the OFF position, the ionization source is shielded on all sides by plastic, so that virtually no alpha particles enter the area of the drops. At the ON position, the plastic shielding is removed and the drop area is exposed to the ionizing alpha particles emitted from the thorium-232. At the SPRAY DROPLET position, the chamber is vented by a small air hole that allows air to escape when oil drops are being introduced into the chamber.

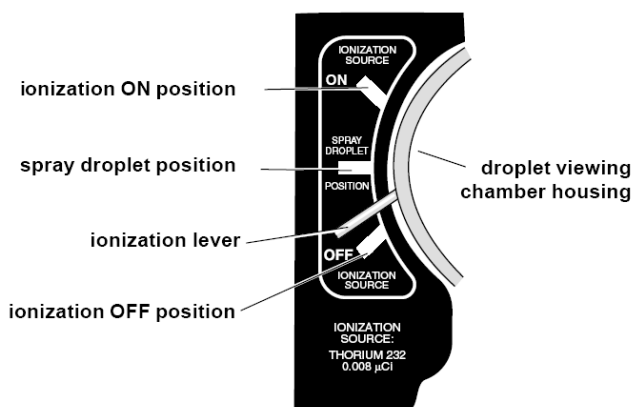


Figure 6. Ionization source lever settings

The plate charging switch has three positions: TOP PLATE -: the negative binding post is connected to the top plate. TOP PLATE +: the negative binding post is connected to the bottom plate. PLATES GROUNDED: plates are disconnected from the high voltage supply and are electrically connected, to ensure no electric field is applied in the droplet chamber.

9. **Introduce oil droplets into the chamber:** Prime the atomizer by giving it a few squeezes into a tissue. Ensure that the tip of the atomizer is pointed down (90° to the shaft; see Figure 7)

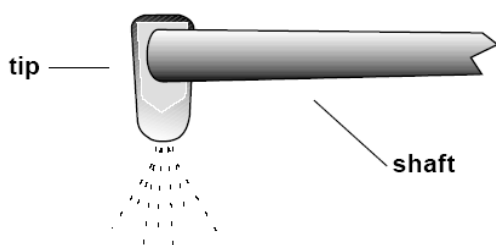


Figure 7. Correct position of the atomizer tip.

Move the ionization source lever to the SPRAY DROPLET position to allow air to escape from the chamber during the introduction of droplets into the chamber. Place the nozzle of the atomizer into the hole on the lid of the droplet viewing chamber. While observing through the viewing scope (or camera image on the computer), squeeze the atomizer bulb with one quick squeeze. Then squeeze it slowly to force the droplets through the hole in the droplet cover, through the droplet entry hole in the top capacitor plate, and into the space between the two capacitor plates. The drops will appear as very fine points of light against the dark background. When you see a shower of drops through the viewing scope, move the ionization source lever to the OFF position.

Note: If repeated “squirts” of the atomizer fail to produce any drops in the viewing area but produce a rather cloudy brightening of the field, the hole in the top plate or in the droplet hole cover may be clogged. Refer to the PASCO manual Maintenance section for cleaning instructions. The exact technique of introducing drops will need to be developed by the experimenter. The object is to get a small number of drops, not a large, bright cloud, from which a single drop can be chosen. The drops are forced into the viewing area by the pressure of the atomizer. Therefore, excessive use of the atomizer can cause too many drops to be forced into the viewing area and, more importantly, into the area between the chamber wall and the focal point of the viewing scope. Drops in this area prevent observation of drops at the focal point at the scope. If the entire viewing area becomes filled with drops, so that no one drop can be selected, either wait three or four minutes until the drops settle out of view, or disassemble the droplet viewing chamber (after turning off the high voltage), which removes the drops.

As a drop is chosen, it may be necessary to focus the microscope slightly. (DO NOT CHANGE THE EYEPIECE-RETICLE FOCUS). Drop selection is discussed in the appendix. From the drops in view, select a droplet that both falls slowly (about  $0.02 - 0.05$  mm/s – major reticle ticks are spaced  $0.5$  mm apart) when the plate charging switch is in the PLATES GROUNDED position, and which has sufficient charge such that its direction of motion may be reversed when the sign of the electric field is changed when the charging switch is flipped. The microscope has special, non-inverting optics, so that a falling drop appears to go down in the field of view and a rising drop appears to go up. Keep the drop in view until most of the mist has cleared from the field of view. If drops move sideways on successive runs, the apparatus is not properly leveled

and should be corrected. If the drift is not excessive, the horizontal angle adjustment can be used for short ranges. This adjustment should be located near the center of its range at the start of the measurement run.

10. The goal of the experiment is to obtain rise and fall time measurements for as many drop and charge combinations as possible. With less data you will not be able to see charge quantization with your data alone. One student should record data on the computer while the other controls the yellow board and the apparatus. Drops should be visible to both students at a lab bench on the computer screen if the camera is used.

The greatest accuracy of measurement is achieved if you time from the instant the bright point of light passes behind the first major reticle line to the instant the bright point of light passes behind the second major reticle line. The lines are 0.5 mm apart.



For each drop, start with a rise time measurement

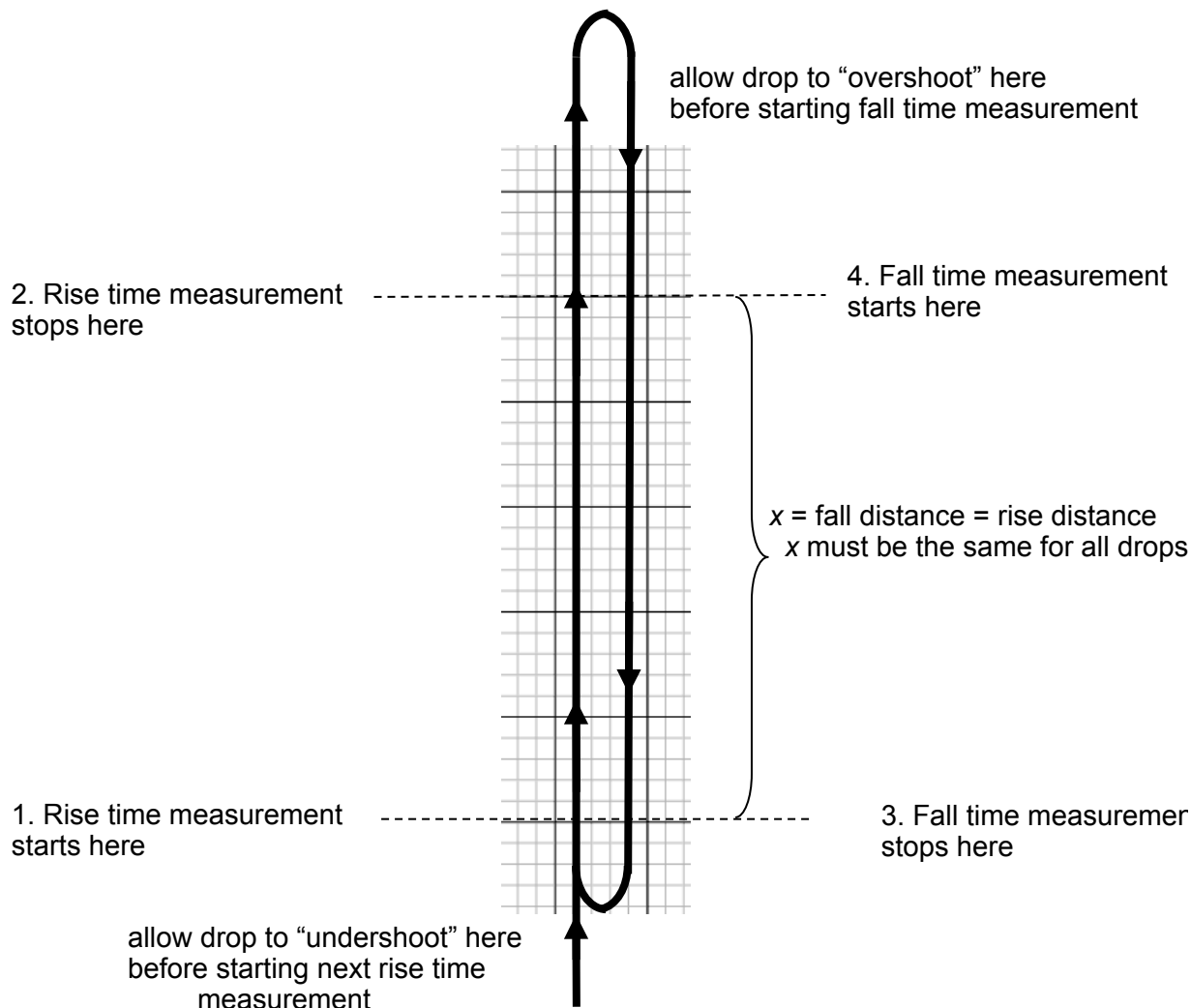


Figure 4. Technique of measuring the rise and fall times. The drop should stay on a vertical path; the return loops are displaced for clarity. Only a portion of the reticle is shown.

- Click on the 'InitDAQSystem' to initialize the program's data arrays.
- After an oil drop is selected, start the measurement sequence with the rise time measurement (i.e. the measurement with the electric field in which the drop appears to fall.)
- The person at the yellow board can press the 'Start' button to measure the rise time.
- The 'Stop' button must be pressed on the yellow board to end that one measurement.
- Use the electric field polarity switch to control the oil drop to use it again.
- You may repeat up to 20 times with a single oil drop - the maximum limit in the program.

- When you are done with an oil drop, click on 'Stop' with the mouse. Save the data in a file. Note the name of your file in your notebook.
- To redo an oil drop, just click on the 'Redo' button with the mouse. Previous data for that particular oil drop will be discarded.
- To continue with the next oil drop, click on 'Next' with the mouse.
- Repeat to a maximum of 20 different oil drops with the 'Timer1' program. The files are small. It is safest to write to a new file after each drop.
- When all data taking is done, click on 'Quit' to exit the program.

11. When sufficient data have been taken on a drop-charge combination, a source of ionizing radiation, Thorium-232, is supplied in the apparatus. This procedure is also useful in case you have problems getting charges on the drop. To change the charge on the drops in the viewing chamber, turn the ionization lever to the ON position briefly until the velocity of your drop changes in an applied electric field. Be sure to flip the ionization lever back to OFF when you are done with it.

12. If the apparatus becomes clogged or dirty during use, it must be cleaned very carefully. Gently remove the cover and the top plate. Blow out any oil in the well and the hole. Use a Kimwipe to absorb excess oil on the two plates. Blow any dust which may remain out of the hole and plates. Replace the cover assembly.

13. At the end of the class period, the apparatus should be cleaned and the 6-conductor cable should be unplugged. Leave the 110-volt line cord at the power supply box plugged. Clean up any oil and discard any dirty Kimwipes.

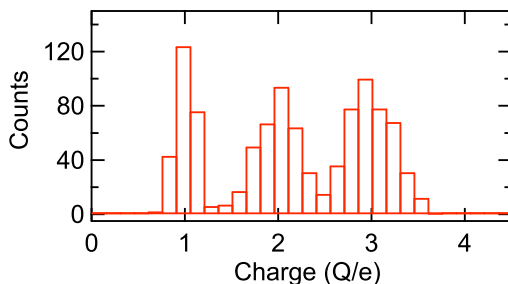
14. In week one you and your lab partner should become comfortable with the apparatus and take data for 3 - 5 drops (rise and fall). You'll write a spread sheet in Origin to analyze these drops using the technique described below and learn to recognize and optimize drops for good data collection.

15. In week two you and your lab partner will take and analyze data for as many drops as possible. At the end, you'll submit your data to be combined with the rest of the class so that you can analyze a larger data set for your report. The larger data set will enable you to resolve the charge quantization more easily..

## VII. Data analysis of individual drops

The expressions need to calculate the charge on a drop are collected in the appendix. The expression for the charge on the drop is given in Eq. 15

$$Q = \left[ \frac{1}{f_c^{3/2}} \right] \left[ \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \right] \left[ \sqrt{\frac{1}{t_g} \left( \frac{1}{t_g} + \frac{1}{t_{rise}} \right)} \right]. \quad (15)$$



**Example of a histogram** – Histograms are used to show the distribution of data values present in a data set. A histogram shows the frequency of occurrence (vertical axis) of a particular range of values within a certain bin width (horizontal axis) that is present in the data set.

In our experiment, we treated each rise-fall event on equal footing with every other event. You may find that the charge state of a given drop changes during successive measurements. For this reason, we treat each rise and fall for a given drop as an independent measurement. We then produce a single column of charge values — each point representing a charge value obtained from a given rise-fall event. A histogram of the data should reveal that the charge values are concentrated on discrete values that are separated by an integer multiple of the electron charge.

The following procedure should be used in analyzing your data. In your report, analyze the data taken by your group as well as the combined data for the entire class. Each group should post their charge values as a single column text file as indicated in the Report section. First, cut and paste the data values from each file posted by your classmates into one file. This file should just be one row of measured charge values for the entire class.

1. Import the data set of measured charge values into Origin: **File → Import → Import Wizard**.
2. Arrange the data in ascending order: **Worksheet → Sort Columns → Ascending**.
3. Plot a histogram the data set: **Plot → Statistics → Histogram**.
4. Change the bin width so that the distribution profile is clearly visible. The bin width can be changed from the plot details dialog box. To open the plot details dialog, double click on the histogram, then choose the details tab. Un-check the **Automatic Binning** box and manually adjust the **Bin Size**.
5. Place this histogram plot in your report.
6. The smaller charge values should show the charge quantization. From the histogram, estimate the largest charge value for which the distribution is well defined, i.e. a clear peak is visible. Only analyze the data that is smaller than this value.

7. From the histogram, pick out the range of values that capture all the points about a given peak, e.g., all the points about  $N=1e$ . Next, find this range of data values from the data column corresponding to the points about a given peak. This should be easy to do since you have sorted the data in ascending order.
8. Highlight this range of values from the column and copy it. Next, create a new column in the workbook by right clicking in the gray area and selecting **Add New Column**. Paste the selected values into the new column.
9. The new column contains the distribution of data values about a given multiple of  $e$ . Highlight this column by left clicking on the column name. Once highlighted, right click and select **Statistics on Columns → Open Dialog**. Under **Plots**, check **Histograms** then hit OK.
10. You should see a histogram of the data subset along with computed mean  $\bar{q}$  and standard deviation  $\sigma$ . Note, the error in the mean decreases as  $\sqrt{N}$ , where  $N$  is the number of points in distribution. Thus, your uncertainty in the mean is  $\bar{q} \pm \sigma/\sqrt{N}$ . Report the mean, standard deviation and the error in the mean for each peak to that you analyze.
11. Repeat 7 – 10 for all peaks to be analyzed.
12. Estimate the value of  $e$  by finding the smallest value of charge which is an integer multiple of measured mean values.

## IX. Report

- A. You should discuss briefly the apparatus and the manner in which you obtained the data.
- B. Put your data in the folder in the P401 common area inside the folder labeled Millikan Oil Drop Experiment. Indicate your last name and the last name of your partner in the filename and

your section number. Example: Jones&Smith-L1.txt. The TAs will distribute the concatenated file of every ones oil drop data.

C. Your report should include a histogram of count vs. charge in units of  $e$  . Provide two histograms: one for your group and a second for the entire class.

D. Make a table indicating the mean, standard deviation, and the standard error in the mean for each peak that is analyzed. Provide two tables: one for your group and a second for the entire class.

E. Report the mean value of the electron as well as the uncertainty for your group and for the entire class.

## Appendix I

### Summary of expressions needed to calculate the charge on a drop

For each drop we measure the fall time,  $t_g$ , and the rise time,  $t_{rise}$ . The charge on the drop is calculated from Eq. 12.

$$Q = \left[ \frac{1}{f_c^{3/2}} \right] \left[ \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \right] \left[ \sqrt{\frac{1}{t_g} \left( \frac{1}{t_g} + \frac{1}{t_{rise}} \right)} \right]. \quad (12)$$

The quantities in this expression are identified in the table below.

quantity	Value
Cunningham correction	$f_c = 1.07$ (typical – varies with drop radius see discussion below)
Plate separation	$d = 3.175 \pm 0.025$ mm (typical)
Voltage difference between plates	$V = 500 \pm 1$ Volts (actual)
Distance over which fall and rise of drop is measured	$x = 1.43 \pm 0.02$ mm (typical)
Viscosity of air	$\eta = 1.8478 \times 10^{-5}$ kg/m·s (25 °C) $\Delta\eta/\Delta T = 0.0048 \times 10^{-5}$ kg / m·s / °C
Density difference	$\rho = \rho_{oil} - \rho_{air}$
Density of oil	$\rho_{oil} = 886$ kg/m <sup>3</sup>
Density of dry air	$\rho_{air} = 1.29$ kg/m <sup>3</sup>
Acceleration due to gravity	$g = 9.801$ m/s <sup>2</sup>

As noted in the table above the density in the drop calculation expression,  $\rho$ , is the difference of the density of oil and air. While the buoyancy is temperature dependent, it is a smallish correction to a small component of the force on the drop, and the density of air provided in the table above may be used.

$$\rho = \rho_{oil} - \rho_{air}$$

The first factor in the expression for  $Q$  is the Cunningham correction. The correction may be calculated from Eq. AII 12

$$\frac{1}{f_c^{3/2}} \approx 1 - \left( \frac{t_g}{\tau_g} \right)^{1/2} \quad (\text{AII.12})$$

where  $\tau_g$  is given by

$$\tau_g = \frac{2\eta x}{\rho g r_c^2}. \quad (\text{AII.1})$$

In Eq. A11  $r_c$  is related to the mean free path for collisions and depends on the ambient pressure. It is given in Eq. 8:

$$r_c [m] = \frac{6.18 \times 10^{-5}}{p [mmHg]}. \quad (8)$$

The drop radius,  $a$ , is determined from the fall time and is given by

$$a \approx \sqrt{\frac{9\eta x}{2\rho g}} \sqrt{\frac{1}{t_g} - \frac{r_c}{2}}. \quad (10)$$

The viscosity of the air is temperature dependent, with the dependence given in the table. The temperature of the air is determined by measuring the resistance of the thermistor in the bottom plate in the droplet viewing chamber. The relationship between temperature and resistance is given in the table below.



**THERMISTOR RESISTANCE TABLE**

<b>°C</b>	<b><math>\times 10^6 \Omega</math></b>	<b>°C</b>	<b><math>\times 10^6 \Omega</math></b>	<b>°C</b>	<b><math>\times 10^6 \Omega</math></b>
<b>10</b>	<b>3.239</b>	<b>20</b>	<b>2.300</b>	<b>30</b>	<b>1.774</b>
<b>11</b>	<b>3.118</b>	<b>21</b>	<b>2.233</b>	<b>31</b>	<b>1.736</b>
<b>12</b>	<b>3.004</b>	<b>22</b>	<b>2.169</b>	<b>32</b>	<b>1.700</b>
<b>13</b>	<b>2.897</b>	<b>23</b>	<b>2.110</b>	<b>33</b>	<b>1.666</b>
<b>14</b>	<b>2.795</b>	<b>24</b>	<b>2.053</b>	<b>34</b>	<b>1.634</b>
<b>15</b>	<b>2.700</b>	<b>25</b>	<b>2.000</b>	<b>35</b>	<b>1.603</b>
<b>16</b>	<b>2.610</b>	<b>26</b>	<b>1.950</b>	<b>36</b>	<b>1.574</b>
<b>17</b>	<b>2.526</b>	<b>27</b>	<b>1.902</b>	<b>37</b>	<b>1.547</b>
<b>18</b>	<b>2.446</b>	<b>28</b>	<b>1.857</b>	<b>38</b>	<b>1.521</b>
<b>19</b>	<b>2.371</b>	<b>29</b>	<b>1.815</b>	<b>39</b>	<b>1.496</b>

## Appendix II

### Cunningham correction

When drop are small, or, equivalently, when the speed of drops through the air is small, Stokes' law must be modified. The drop is considered small when the ratio of the mean free path for collisions in air to the radius of the drop is no longer much less than one. The correction is semi-empirical and is given as

$$f_c = 1 + A \frac{\ell}{a} + B \frac{\ell}{a} e^{-C \frac{a}{\ell}}. \quad (\text{AII.1})$$

In this expression  $a$  is the radius of the drop, which in the discussion above we have also called the corrected radius. The first term was calculated in kinetic theory by Cunningham.<sup>6</sup> The second term is empirical. The dimensionless constants  $A$ ,  $B$ , and  $C$  are taken from Oki et al.<sup>7</sup>

$$A = 1.246 \quad B = 0.42 \quad C = 0.87. \quad (\text{AII.2})$$

The mean free path,  $\ell$ , is inversely proportional to ambient pressure.

$$\ell [m] = 6.53 \times 10^{-8} \frac{760}{p [\text{mm Hg}]}. \quad (\text{AII.3})$$

The expression above gives the mean free path in meters when the pressure is entered in mmHg. The mean free path at a pressure of 760 mm Hg (standard atmosphere) is  $6.53 \times 10^{-8}$  m. We find for the drops that we choose that the first term in the Cunningham correction is adequate. We then use for the Cunningham correction

$$f_c \approx 1 + A \frac{\ell}{a} \equiv 1 + \frac{r_c}{a}. \quad (\text{AII.4})$$

The factor,  $r_c = A\ell$ , and depends on ambient pressure. Using the constants given above, we find that

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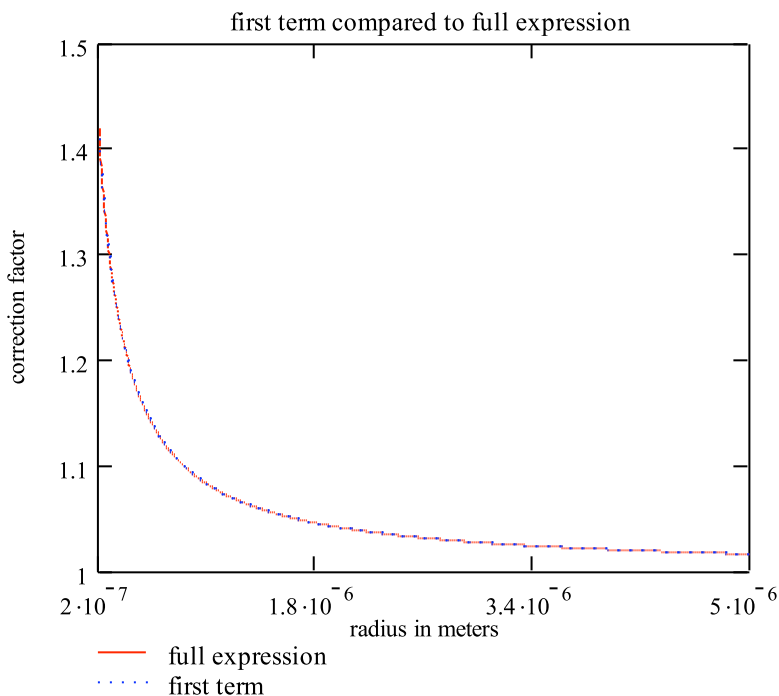
<sup>6</sup> M. Cunningham, Proc. Roy. Soc. (London), 83,357 (1910).

<sup>7</sup> Y. Oki, A. Endo and K. Kondo. See also W. Demarcus and J. Thomas ORNL-1431 (1952).

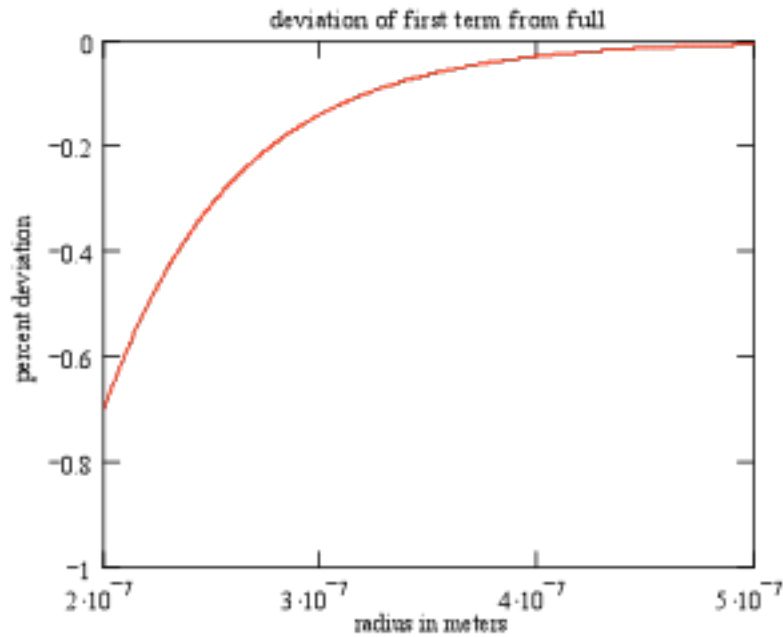
$$r_c [m] = \frac{6.18 \times 10^{-5}}{p [\text{mm Hg}]} \quad (\text{AII.5})$$

In the above expression  $r_c$  is obtained in meters when the pressure is entered in  $\text{mmHg}$ .

In our experiment the radius of the drops is of the order of  $10^{-6}$  m. The graph below compares the first term of the Cunningham correction to the entire expression for the range of drop radii that is used in our experiment. It is not possible to discern a difference between the first term and the complete expression.



The deviation of the first term from the full expression is shown in the graph below for radii less than 0.2 microns. The graph shows that the first term is completely adequate for radii greater than 0.2 microns.



For a typical drop radius of 1 micron, the Cunningham correction is ~1.1. Without this correction our calculation of the electron charge would be incorrect by about 10%. The correction cannot be ignored.

It is useful to develop approximate expressions for the (corrected) drop radius,  $a$ , and related factors. An expression for the (corrected) drop radius is given above in Eq. 10 and repeated below.

$$a = \sqrt{a_o^2 + (r_c/2)^2} - r_c/2 \approx a_o - r_c/2 = \sqrt{\frac{9\eta}{2g\rho} \frac{v_g}{\rho}} - \frac{r_c}{2}. \quad (10)$$

The next term in expansion of the square root gives

$$a \approx a_o \left( 1 - \frac{1}{2} \frac{r_c}{a_o} + \frac{1}{8} \left( \frac{r_c}{a_o} \right)^2 \right). \quad (\text{AII.7})$$

The higher order term is <0.1% for typical drops so it can be ignored.

To calculate the charge on the drop, we need  $1/f_c^{3/2}$  as seen in Eq. 12 above.

$$Q = \left[ \frac{1}{f_c^{3/2}} \right] \left[ \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \right] \left[ \sqrt{\frac{1}{t_g}} \left( \frac{1}{t_g} + \frac{1}{t_{rise}} \right) \right]. \quad (12)$$

With the approximation for  $f_c$  given in Eq. AII.4 above, the factor that appears in the expression for the charge on the drop becomes

$$\frac{1}{f_c^{3/2}} \approx \frac{1}{\left(1 + \frac{r_c}{a_o}\right)^{3/2}} \approx 1 - \frac{3}{2} \frac{r_c}{a_o}. \quad (\text{AII.8})$$

The next term in expansion gives

$$\frac{1}{f_c^{3/2}} \approx 1 - \frac{3}{2} \frac{r_c}{a_o} + \frac{9}{8} \left( \frac{r_c}{a_o} \right)^2 \quad (\text{AII.9})$$

and is again 0.1% for typical drops.

Recall that the uncorrected radius,  $a_o$ , is determined from the fall time measurements. Eq. 8 from above can be written as

$$a_o = \sqrt{\frac{9\eta x}{2\rho g}} \sqrt{\frac{1}{t_g}}. \quad (\text{AII.9})$$

Then Eq. AII.6 can be written as

$$\frac{1}{f_c^{3/2}} \approx 1 - \frac{3}{2} \frac{r_c}{a} = 1 - \left( \frac{\rho g r_c^2}{2\eta x} \right)^{1/2} \sqrt{t_g} \quad (\text{AII.10})$$

The constants that multiply the square root of the fall time have dimensions of time, and it is useful to define

$$\tau_g = \frac{2\eta x}{\rho g r_c^2} \quad (\text{AII.11})$$

so the correction factor is compactly written as

$$\frac{1}{f_c^{3/2}} \approx 1 - \left( \frac{t_g}{\tau_g} \right)^{1/2} \quad (\text{AII.12})$$

We recall that the correction factor, since it depends on the drop radius, is different for each drop.

### Appendix III

#### Calculation of the time to reach terminal velocity

In the above development we assumed that the velocity of the drop was constant, i.e. it had reached its terminal velocity. Suppose we solve Eq. 1 in the case of no electric field.

$$\vec{F} = m \frac{d}{dt} \vec{v} = \vec{F}_g + \vec{F}_{drag} = -mg\hat{z} - K \vec{v}. \quad (\text{AIII.1})$$

This is recognized as an inhomogeneous, linear, first order differential equation,

$$\frac{d}{dt} \vec{v} + \frac{K}{m} \vec{v} = -g\hat{z}. \quad (\text{AIII.2})$$

The homogeneous equation is easily integrated and the inhomogeneous solution is a constant. We choose the initial condition  $\vec{v}(0) = 0$ . The solution is

$$\vec{v}(t) = -\frac{mg}{K} (1 - e^{-Kt/m}) \hat{z}. \quad (\text{AIII.3})$$

The characteristic time is seen to be  $m/K = 2\rho a^2/9\eta \approx 10^{-5} s$  for a drop with a radius of  $10^{-6} m$ . Thus the drop very quickly comes to its terminal velocity.

Eq. AIII.3 can be integrated to give the drop position as a function of time. Writing  $v = dz/dt$ , Eq. AIII.3 becomes

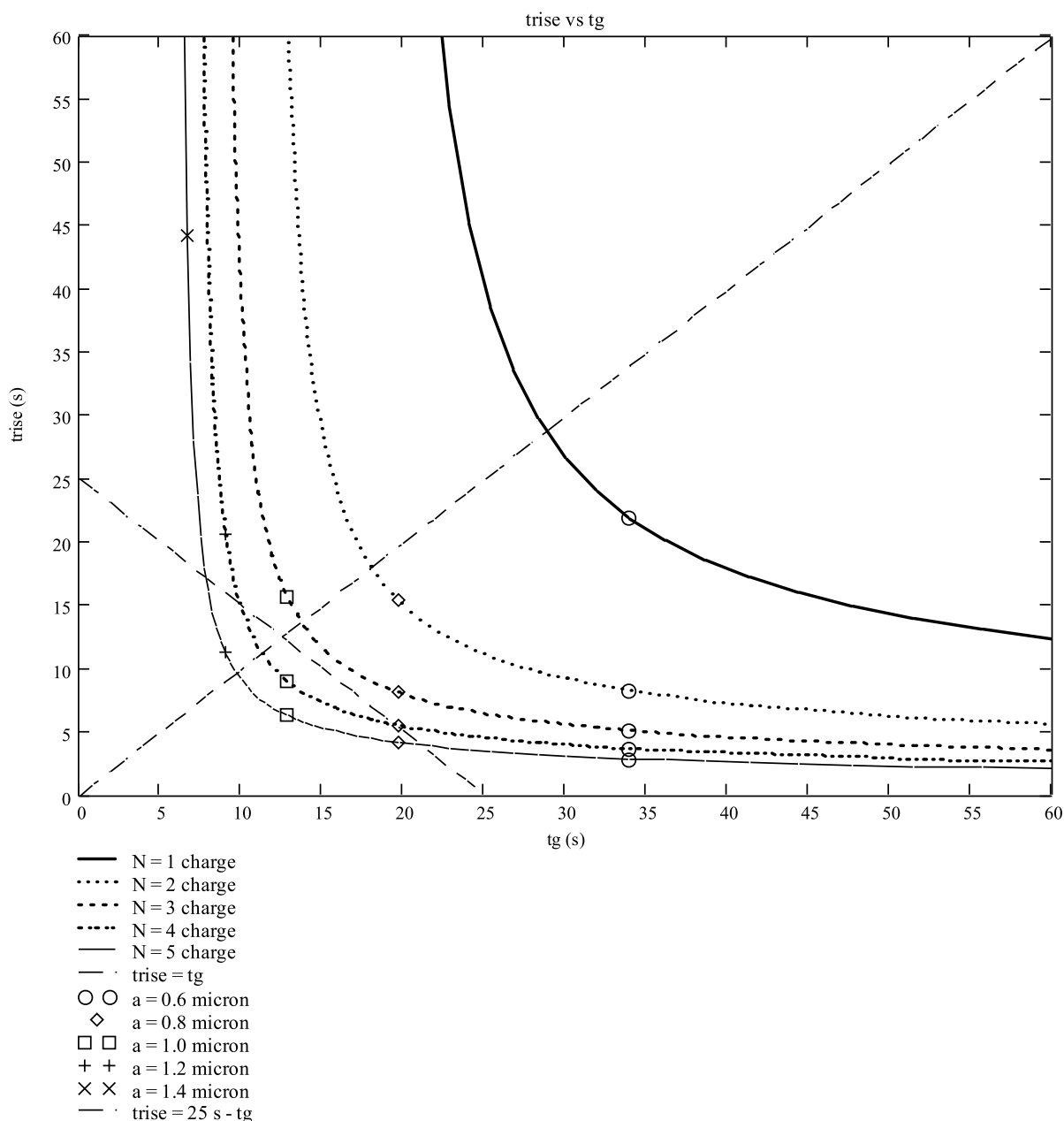
$$\frac{dz}{dt} = -\frac{mg}{K} (1 - e^{-Kt/m}). \quad (\text{AIII.4})$$

which is an inhomogeneous, linear, first order differential equation. We choose the initial condition  $z(0) = 0$ . The solution is

$$z(t) = -\frac{mg}{K} \left[ t - \frac{m}{K} (1 - e^{-Kt/m}) \right] \rightarrow -\frac{mg}{K} \left[ t - \frac{m}{K} \right]. \quad (\text{AIII.5})$$

The solution is recognized as describing uniform motion.

The motion of the drop through air is actually a problem in fluid dynamics. For the velocity of the drop to change, it must impart kinetic energy to the fluid. Thus the fluid resists the motion of the drop. This effect is known as the backflow correction and is treated by adding



to the mass of the drop one-half of the mass of the displaced fluid.<sup>8</sup> In Eq. AIII.1 the inertial mass on the right-hand side is taken to be

<sup>8</sup> See, for example, L. M. Milne-Thomson, *Theoretical Hydrodynamics*, 3<sup>rd</sup> edition, §15.32.



$$\vec{F} = \left( m + \frac{1}{2} m_{air} \right) \frac{d}{dt} \vec{v} = \left( m + \frac{1}{2} \frac{4\pi}{3} \rho_{air} a^3 \right) \frac{d}{dt} \vec{v} . \quad (\text{AIII.6})$$

With this addendum the time constant in Eq. AIII.3 becomes

$$\vec{v}(t) = -\frac{mg}{K} \left( 1 - e^{-Kt/(m+\frac{1}{2}m_{air})} \right) \hat{z} . \quad (\text{AIII.7})$$

In this experiment we measure the drop when it is uniform motion so the consideration of the backflow correction is not important.

## Appendix IV

### The importance of drop selection

The graph below demonstrates the importance of proper drop selection. The graph shows a calculation of drop rise time,  $t_{rise}$ , versus drop fall time,  $t_g$ , for typical parameters of our experiment. There is a lot on the graph, but it is very useful to understand it. The fall distance,  $x$ , in the calculation is 1.43 mm. This distance corresponds to just under three large divisions on the scale of one of the Millikan set-ups. The other parameters in the calculation, voltage on the plates, the plate separation, and the oil density and viscosity, are the same as in our experiment. Recall that the fall time depends on the drop radius. The rise time also depends on the radius, but the rise time depends on the charge on the drop. We would like to select drops with one, two, three, etc. charges to see charge quantization convincingly. The solid curve in the graph above shows  $t_{rise}$  versus  $t_g$  for drops with one charge. Each point on the curve has a different drop radius. The drop radius varies between  $\sim 0.5$  and  $\sim 0.7$  microns on this curve. The combination of  $t_{rise}$  and  $t_g$  for which the drop radius is 0.6 microns is indicated by a circle. Note that on this curve the fall time,  $t_g$ , is always greater than 25 s. If we want to see a drop with one charge, we must select drops with a fall time greater than 25 s.

The dotted curve shows  $t_{rise}$  versus  $t_g$  for drops with two charges. On this curve the drop radius varies between  $\sim 0.5$  and  $\sim 0.9$  microns. In addition to the combination of  $t_{rise}$  and  $t_g$  for which the drop radius is 0.6 microns, indicated by a circle, the combination for which the drop radius is 0.8 microns is indicated by a diamond. Note that on this curve the fall time,  $t_g$ , is always greater than 12 s. Same message: if we want to see a drop with two charges, we must select drops with a fall time greater than 12 s.

We measure the rise and fall times to a certain precision. We could image including timing errors in the graph by changing the sharp lines to fuzzy lines which define “banana” shaped areas. The ends of the “bananas” would be close to one another. The greatest separation between the “bananas” is on the line  $t_{rise} = t_g$ . This line is indicated in the graph by a dashed line that goes through the origin. It is easy to see that the separation of the curves is greatest on this diagonal line.

A second dashed line does between the two points on the axes,  $(t_{rise}, t_g) = (0, 25 \text{ s})$  and  $(t_{rise}, t_g) = (25 \text{ s}, 0)$ .  $(t_{rise}, t_g)$  combinations between this dashed line and the origin have  $t_{rise} < 25 \text{ s}$  and  $t_g < 25 \text{ s}$ . If we select such drops, we will never see one, two or three charges.

The graph shows that drop selection is crucial for a convincing demonstration of charge quantization. Select a slowly falling drop. Then see if its rise time is similar to the fall time. Such drops should produce the most satisfying results.